

TOPOLOGY - QUAL EXAM, FALL 2021

The Topology - Qual Exam, Fall 2021 is on Gradescope at 1:00 - 4:00 pm, San Diego local time, Thursday, September 9, 2021. After you finish, you will have 15 more minutes to upload your solutions, as pdf files, on Gradescope. You should assign pages for each problem on Gradescope. This exam on Gradescope will close at 4:15pm, San Diego local time. It is your responsibility to make sure that you have a working internet, but in case you encounter unexpected issues to upload your solutions on Gradescope, you may email them to me (xuzhouli@ucsd.edu).

This exam won't be proctored. You are NOT allowed to use books or notes during the exam. The instructor reserves the right to conduct an oral follow-up exam if he notices a possible academic integrity violation.

1. (45 pts) Let $\mathbb{R}P^2$ be the real projective plane. Let $T = S^1 \times S^1$ be the torus.
 - (a) (5 pts) Let A be their connected sum. Provide a presentation of the group $\pi_1(A)$.
 - (b) (5 pts) Let $B = \mathbb{R}P^2 \vee T$ be their wedge sum (one-point union). Provide a presentation of the group $\pi_1(B)$.
 - (c) (5 pts) Compute $H_*(A; \mathbb{Z})$.
 - (d) (5 pts) Compute $H_*(B; \mathbb{Z})$.
 - (e) (5 pts) Compute $H^*(A; \mathbb{Z}/2)$ as a ring.
 - (f) (5 pts) Compute $H^*(B; \mathbb{Z}/2)$ as a ring.
 - (g) (5 pts) Is A orientable? Explain your reason.
 - (h) (5 pts) Is B homotopy equivalent to a manifold? Explain your reason.
 - (i) (5 pts) Let C be a double cover (2-sheeted covering space) of B . Compute the Euler characteristic of C .

2. (25 pts) Let $\mathbb{C}P^2$ be the complex projective plane.
- (a) (5 pts) Compute $\pi_1(\mathbb{C}P^2)$.
 - (b) (10 pts) Could $\mathbb{C}P^2$ be an n -sheeted covering space ($n > 1$) of another space D ? If so, compute $H_*(D; \mathbb{Z})$. If not, explain your reason.
 - (c) (5 pts) Compute $H_*(\mathbb{C}P^2 \times \mathbb{R}P^2; \mathbb{Z})$.
 - (d) (5 pts) Compute $H_*(\mathbb{C}P^2 \times \mathbb{R}P^2; \mathbb{Z}/2)$.
3. (10 pts) Let X be a 3-dimensional simply-connected closed manifold (compact, no boundary). Show that X is homotopy equivalent to S^3 .
4. (10 pts) Let K be the space $\{[x, y, z, w] \in \mathbb{C}P^3 : x^4 + y^4 + z^4 + w^4 = 0\}$. It is known that K is a simply-connected closed manifold with Euler characteristic 24. Compute $H_*(K; \mathbb{Z})$.