# Qualifying Exam 281 A&B

**Directions and Rules of Conduct**

You can use Theorems from the lectures and books by Aart Van de Vaart or Martin Wainwright to support your claims. If a theorem from these two books is to be used, a student must provide written evidence as to why and how are the conditions of that theorem satisfied. Exceptions to this rule are theorems of real analysis (such are continuous mapping theorem, dominated convergence theorem, etc...). Other results cannot be used unless proved.

All solutions and submissions are subject to Academic Integrity principles of UCSD therefore indicating, among other things, that all submissions are independent and a result of an individual work of a student. All violations will be reported to the Academic Senate.

For the duration of the exam, Professor Jelena Bradic can be reached via email jbradic@ucsd.edu. All questions and concerns need to be addressed to the Professor's email directly. Expect ample time in obtaining the response from any query.

Exam needs to be submitted to Professor Bradic directly by a strict deadline of 8:15pm of September 13th. Please allow enough time to upload your solutions to Canvas; a good one hour should be allocated for this purpose alone.

To complete the exam complete two problems. Therefore the complete exam will have Problem 1+2 or Problem 1+3 or Problem 2+3 structure.

Total number of points is 70 = 35 (Problem 1 or 2 or 3 ) + 35 (Problem 1 or 2 or 3).

Name (Printed): _____

Student ID Number: _____

Signature: _____

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## Problem 1 [35 points]

(a) [ 22 points]

Compute the Gaussian complexity of a $l_q$, $q \in [1, \infty)$ unit ball in $\mathbb{R}^d$. Here, for any vector $\theta \in \mathbb{R}^d$, $\|\theta\|_q^q = \sum_{j=1}^d |\theta_j|^q$.

(b) [ 13 points]

Work out the case of $q = \infty$.

## Problem 2 [35 points]

Solve Exercise 13.6 page 450., from the High-Dimensional Statistics textbook by Martin J. Wainwright.

## Problem 3 [35 points]

Solve Problem 24 page 84., from the Asymptotic Statistics textbook by A. W. van der Vaart.

## QUAL EXAM: MATH 281C – FALL 2021

- *Define any symbol you use unless its meaning is clear from context. Name any result you use if it has a name. Be concise and clear.* **Justify all your answers**.

3. PROBLEM 3. (LR TEST, RAO'S SCORE TEST AND WALD'S TEST)

Suppose that $X_1, \ldots, X_n$ are i.i.d. from the Weibull p.d.f. $\theta^{-1}\gamma x^{\gamma-1} e^{-x^\gamma/\theta} I_{(0,\infty)}(x)$, where $\gamma > 0$ and $\theta > 0$ are unknown. Consider the problem of testing $H_0 : \gamma = 1$ versus $H_1 : \gamma \neq 1$.

(a) Find an LR (likelihood ratio) test, and discuss whether the asymptotic chi-square approximation theorem can be applied.

(b) Find the forms of $W_n$ for Wald's test, and $R_n$ for Rao's score test.