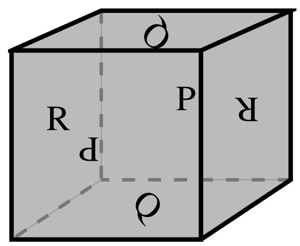
Fall 2017

1. Let $G = \langle a, b \rangle$ be a free group on two generators, let H be the subgroup generated by the elements

$$(a^2b)^3 \quad (ab)^3 \quad bab \quad a^2ba^{-1},$$

and let $N(H)$ be the normaliser of H in G. By considering the deck translations of a suitable covering space, identify the quotient group $N(H)/H$.

2. Let X be the space obtained by gluing opposite pairs of faces of a standard cube I^3 via 180 degree rotations, as shown. Compute the homology $H_*(X; \mathbb{Z})$.



3. Let X be the space obtained by gluing the two ends of $S^2 \times I$ via the antipodal map of S^2. Compute its homology $H_*(X; \mathbb{Z})$.

4. Let X be a path-connected space whose homology groups in positive dimensions are $H_k(X; \mathbb{Z}) = \mathbb{Z}/k\mathbb{Z}$. Compute the integer homology $H_*(\mathbb{R}P^2 \times X; \mathbb{Z})$.

5. Show that there exists a degree 1 map from $T^3 = S^1 \times S^1 \times S^1$ to S^3, but not vice versa.

6. Let M be a closed oriented 4-manifold whose second homology $H_2(M; \mathbb{Z})$ has rank 1. Show that there does not exist a free action of the group $\mathbb{Z}_2$ on M.

7. Show that a closed, compact, simply-connected 3-manifold M^3 is homotopy-equivalent to S^3.

8. Let P be the Poincaré homology sphere, a 3-manifold whose fundamental group has order 120 and whose universal cover is S^3. Compute π_3 of the one-point union $P \vee S^3$.