Topology Qualifying exam, Spring 2006

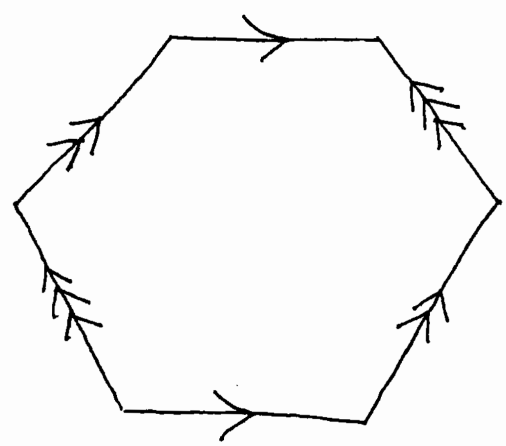
You have three hours to answer these questions. No notes or books are allowed. If you are using a well known theorem, please recall and indicate a reference. There are 5 questions of varying difficulty in all, you are not expected to be able to solve all of them. All the best.

1. Let X be the topological space obtained by taking a regular hexagon and identifying opposite edges in a parallel fashion as shown.

   a) Calculate the integral cohomology ring of X.

   b) Can X be homotopy equivalent to a two dimensional compact manifold?

   c) Calculate the fundamental group of X.



2. Let Σ_n denote the Riemann surface of genus n. Use the Euler characteristic to show that there is no finite covering map from Σ_{n+1} to Σ_n for $n > 2$.

3. Let M be an $2n+1$ dimensional compact oriented manifold with $\pi_1(M) = \mathbb{Z}/k$, where k is an odd integer. Show that the degree of any map from M to $\mathbb{RP}^{2n+1}$ is an even integer.

4. Let X be a CW complex with one 0-cell, one 1-cell, two 2-cells and one 4-cell. Assume that the attaching maps of the two 2-cells to the 1-cell have degree 2 and 4 respectively.

   a) Calculate the Euler characteristic of X.

   b) Calculate the integral and mod 2 homology of $X \times \mathbb{RP}^2$.

5. Let $X(n,k) = \mathbb{RP}^n/\mathbb{RP}^k$ denote the quotient space of $\mathbb{RP}^n$ obtained by identifying $\mathbb{RP}^k$ to a point for $0 < k < n$. Calculate the mod 2 cohomology ring of $X(2k+2, k)$.