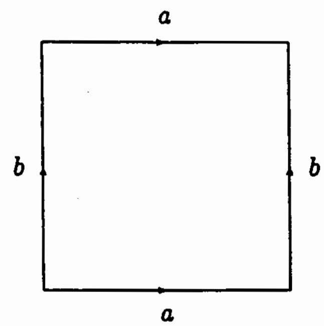
TOPOLOGY

Qualifying Exam **Spring 2002**

1. Let G be a group of homeomorphisms acting freely on S^{2n} so that for all $g \in G$ $gx = x$ for some x if and only if $g = 1$. Prove $|G| \leq 2$.

2. Let $f : S^n \longrightarrow S^n$ be a continuous function such that $f(-x) = f(x)$. Prove $\deg f$ is even.

3. Consider the following space X obtained by identifying the edges of the unit square in the following manner:



   (a) Compute $H_*(X; \mathbf{Z})$ and $H^*(X; \mathbf{Z})$.
   (b) Prove X is an orientable manifold.
   (c) Compute the ring structure of $H^*(X; \mathbf{Z})$.

4. Let $p : E \longrightarrow X$ be a covering space.
   (a) If X is a manifold prove E is also.
   (b) If X is a topological group, sketch a proof that there is a multiplication map $m : E \times E \longrightarrow E$ such that $pm = \mu(p \times p)$ where $\mu : X \times X \longrightarrow X$ is the multiplication on X.
   (c) If X is a cell complex, prove E is also.

5. Let $S^n \xrightarrow{f} \mathbf{RP}^n$ be the covering space map. Prove f is not null homotopic.

6. Let X be an n-dimensional manifold and X^0 its R-orientation sheaf. Give R the discrete topology. If X is R-orientable prove X^0 is homeomorphic to $X \times R$.

7. If all n-fold cup products vanish on $H^*(Y)$ and $f : X \longrightarrow Y$ is a continuous map, prove all $n+1$ fold cup products vanish in $H^*(Cf)$ where Cf is the mapping cone of f.

8. Let M be an n-dimensional compact connected orientable manifold. Let $\zeta_M \in H_n(M; \mathbf{Z})$ be the fundamental class. Suppose $f : S^n \longrightarrow M$ is a continuous function with $f_*(\zeta_s) = \zeta_M$. Prove $H_*(S^n; \mathbf{Z}) \cong H_*(M; \mathbf{Z})$.

9. Let $S^1 \vee S^1$ be the one point union of circles. Prove $\pi_1(S^1 \vee S^1, x_0)$ is not abelian.

10. Let $p : E \longrightarrow X$ be a covering space and let $C \subseteq E$ be a connected component of E. prove $p_{|C} : C \longrightarrow X$ is also a covering space.