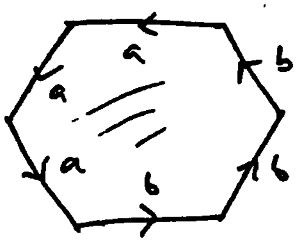
2003 Fall Topology Qual

Two and a half hour exam. Each question is worth five marks. You can use any standard facts or theorems in your work provided you state them clearly.

1. *Commensurability* is the equivalence relation on spaces generated by saying that $X \sim Y$ if X is a finite cover of Y (or vice versa). What are the commensurability classes of closed (not necessarily orientable) 2-dimensional surfaces?

2. Let $X = S^1 \vee S^1$ be the figure-of-eight space. Draw pictures of the covers of X corresponding to the subgroups $\langle abab \rangle$ and $\langle ab, ba \rangle$.

3. Let X be the space obtained by identifying the edges of a solid hexagon as shown below. Compute $H_*(X; \mathbb{Z})$.



4. Let N be submanifold of S^3 which is homeomorphic to a thickened torus $T^2 \times I$. Let X be its exterior, that is the closure of $S^3 - N$. Use Mayer-Vietoris to compute the homology $H_*(X; \mathbb{Z})$.

5. Let M^4 be a closed connected simply-connected 4-manifold. Show that $H_1(M; \mathbb{Z}) = H_3(M; \mathbb{Z}) = 0$ and that $H_2(M; \mathbb{Z})$ is a free abelian group.

6. Compute $\mathrm{Tor}(\mathbb{Z} \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_8, \mathbb{Z} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_4)$.

7. Consider the standard embedding $\mathbb{C}P^1 \subseteq \mathbb{C}P^2$. Show that any map $f : S^2 \to \mathbb{C}P^2$ whose image $f(S^2)$ is disjoint from $\mathbb{C}P^1$ must be null-homotopic.

8. Describe the universal cover of $X = \mathbb{R}P^3 \vee S^2$, and use it to compute the abelian group $\pi_2(X)$.